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Particle Filter Based Ownship Evasive Maneuver under Torpedo Attack

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Abstract— Particle filter is proposed for tracking a torpedo using bearings-only measurements when torpedo is attacking an ownship. Towed array is used to generate torpedo bearing measurements. Ownship evasive maneuver is used for observability of the bearings-only process. Particle filter combined with Modified Gain Bearings-Only Extended Kalman Filter is used to estimate torpedo motion parameters, which are used to calculate optimum ownship evasive maneuver. Monte-Carlo simulation is carried out and the results are presented for typical scenarios.

Index Terms— Evasive Maneuver, Modified Gain Bearings-only Extended Kalman Filter, Monte-Carlo Simulation, Observability, Particle Filter, Torpedo Motion Parameters, Towed array .

1 Introduction

IN the ocean environment, two dimensional bearings-only target motion analysis is generally used. An ownship monitors noisy sonar bearings from a radiating target and finds out target motion parameters (TMP) - viz., range, course, bearing and speed of the target. The basic assumptions are that the target moves at constant velocity most of the time. The ownship motion is unrestricted. The target and ownship are assumed to be in the same horizontal plane. The problem is inherently nonlinear as the measurement is nonlinear. Bearings-Only Tracking (BOT) is the determination of the trajectory of a target solely from bearing measurements. In this passive target tracking, a single ownship monitors a sequence of bearing measurements, which are assumed to be available at equi - spaced discrete times. The target motion analysis can be viewed as target localization and its tracking. The BOT area has been widely investigated [1-4] and numerous solutions for this problem have been proposed.

Maximum Likelihood Estimator (MLE) is found to be a suitable algorithm for passive target tracking applications, by virtue of its characteristics [1]. This is gradient search based on a batch processing of all the available measurements. MLE is asymptotically efficient, consistent, unbiased and its covariance matrix approaches the Cramer-Rao bound for large samples. Instead of assuming some arbitrary values, PLE outputs are used for the initialization of MLE [8].

Another approach, utilization of Extended Kalman Filter (EKF) in modified polar [MP] coordinates [9] frame is found to be useful for this nonlinear application. In this algorithm, the observable and unobservable components of the estimated state vector are automatically decoupled. Such decoupling is shown to prevent covariance matrix ill- conditioning, which is the primary cause of instability. The MP state estimates are asymptotically unbiased. A hybrid coordinate system approach developed by Walter Grossman is also another successful contribution to bearings-only passive target tracking [10].

Another successful contribution to this field is by Song & Speyer [11]. The divergence in EKF [3, 4] is eliminated by modifying the ownship gains. This algorithm is named as modified gain bearings-only extended Kalman filter (MGBEKF). The essential idea behind MGBEKF is that the nonlinearities be “modifiable”. This algorithm has some similarities with the pseudo measurement function but it is not the same. In pseudo measurement filter, the gain is a function of past and present measurements. It is to be noted that MGBEKF is based on the algorithm for the EKF, the gain of the MGBEKF is a function of only past measurements. So, by eliminating the direct correlation of the gain and measurement noise process in the estimates of MGBEKF, the bias in the estimation is eliminated. A simplified version of the modified gain function is made available by Galkowski and Islam [12]. This version is useful for air applications, where elevation and bearing measurements are available. It is further modified for underwater target tracking applications [13-14], where bearings-only measurements are available.

The traditional Kalman filter is optimal when the model is linear. Unfortunately, many of the state estimation problems like tracking of the target using bearings only information are nonlinear, thereby limiting the practical usefulness of the Kalman filter and EKF. Hence, the feasibility of a novel transformation, known as unscented transformation, which is designed to propagate information in the form of mean vector and covariance matrix through a nonlinear process, is explored for underwater applications. The unscented transformation is coupled with certain parts of the classic Kalman filter. It is easier to implement and uses the same order of calculations. Using bearings-only measurements, Unscented Kalman filter (UKF) algorithm estimates target motion parameters [15-16]. UKF can be treated as an alternative to MGBEKF. But still, the basic constraint that is the PDF of noise in the measurements is to be Gaussian, for optimum results. Hence UKF can take up

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nonlinearity but not non-Gaussian noise in the measurements.

Particle Filters (PF) [17]-[19] are the new generation of advanced filters, which are useful for nonlinear and non-Gaussian applications. PF or Sequential Monte-Carlo (SMC) methods use a set of weighted state samples, called particles, to approximate the posterior probability distribution in a Bayesian setup. At any point of time, the set of particles can be used to approximate the PDF of the state. As the number of particles increase to infinity, the approximation approaches the true PDF. They provide nearly optimal state estimates in the case of nonlinear and non-Gaussian systems, unlike Kalman filter based approaches. Because PFs do not approximate nonlinearities or non-Gaussian noise in the system and use a large number of particles, they tend to be computationally complex. However, with the currently available advanced microprocessors, the computation can be easily managed. PF combined with MGBEKF (PFMGBEKF) is proposed in this paper for passive bearings-only torpedo tracking using towed array measurements.

The task is to estimate the torpedo motion parameters, while ownship is in attack by a torpedo. After getting the first contact of the torpedo, ownship tries to escape by doing a certain manoeuvre. This manoeuvre is based on 70^0 relative bearing method, which is being used by Navy. (Details are given in Appendix-A). Here this first manoeuvre is called as ownship safety manoeuvre. The idea is to escape from the field as early and quick as possible. In general, the ownship tries to increase the speed after turning to the required course. This is required for the ownship to escape from the target as early as possible.

The ownship's subsequent escape manoeuvres can be carried out in systematic way, if torpedo's range, bearing, course and speed are known. As these are not available, these are estimated using PFMGBEKF. Here as bearings are only available, ownship safety manoeuvre will be used for observability of the process. During safety manoeuvre, ownship tries to escape in such a way that range between ownship and target becomes maximum value with increase in time. But for getting solution, it is other way round. Range should decrease to get more bearing rate with increase in time. With this constraint, ownship tries to estimate the torpedo motion parameters to calculate proper evasive manoeuvres using Closest Path of Approach (CPA) at various time instants and escape from torpedo attack.

Section 2 describes mathematical modeling of measurements, PFMGBEKF and CPA. PFMGBEKF is developed and implemented on PC platform using MATLAB. Section 3 describes about implementation aspects of the algorithm. Extensive simulation is carried out and the results are presented for three scenarios. Section 4 covers the limitations of the algorithm and finally the paper is concluded in section 5.

2 Mathematical Modeling

2.1 State and Measurement Equations

Let the target state vector be Xs (k) where **Error! Reference**

$$X_s(k) = \left[\dot{x}(k) \quad \dot{y}(k) \quad R_x(k) \quad R_y(k)\right]^T$$

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(1)

where $\dot{x}(k)$ and $\dot{y}(k)$ are target velocity components and **Error! Reference source not found.**are range components respectively. The target state dynamic equation is given by

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma\omega(k)$$

(2)

where Φ and b are transition matrix and deterministic vector respectively. The transition matrix is given by

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix}$$

Error! Reference source not found. (3)

where t is sample time.

$$b(k+1) = \left[0 \quad 0 \quad -\{x_o(k+1) - x_o(k)\} \quad -\{y_o(k+1) - y_o(k)\}\right] \quad (4)$$

$$\Gamma = \begin{bmatrix} t & 0 & t^2/2 & 0 \\ 0 & t & 0 & t^2/2 \end{bmatrix}$$

Error! Reference source not found. (5)

where $x_0(k)$ and $y_0(k)$ are ownship position components. The plant noise ω(k) is assumed to be zero mean white Gaussian with covariance $E[\omega(k)\omega'(k)] = Q\delta_{kj}$ (6)

where

$$Q = \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/8 & 0 \\ 0 & t^3/2 & 0 & t^4/8 \end{bmatrix}$$

Error! Reference source not found.(7)

True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement **Error! Reference source not found.** is modeled as

$$B_m(k+1) = \tan^{-1}\left(r_x(k+1)/r_y(k+1)\right) + \varsigma(k)$$

(8)

where **Error! Reference source not found.** is error in the measurement and this error is assumed to be zero mean Gaussian with variance**Error! Reference source not found.**. The measurement and plant noises are assumed to be uncorrelated to each other. Eqn. (8) is a nonlinear equation and is linearized by using the first term of the Taylor series for**Error! Reference source not found.**. The measurement matrix is obtained as **Error! Reference source not found.Error! Reference source not found. Error! Reference source not found.**

$$H(k+1) = \left[0 \quad 0 \quad \hat{R}_y(k+1/k)/\hat{R}^2(k+1/k) \quad -\hat{R}_x(k+1/k)/\hat{R}^2(k+1/k)\right] \quad (9)$$

since the true values are not known, the estimated values of **Error! Reference source not found.** are used in eqn. (9).

2.2 Particle Filter

The Particle Filter is a statistical brute-force approach to estimation that often works well for problems (i.e., systems that are highly nonlinear) that are difficult for the

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conventional Kalman filter. Let us derive the basic idea of the PF. It was invented to numerically implement the Bayesian estimator. The main idea is intuitive and straight forward. At the beginning of the estimation problem, we randomly generate N state vectors based on the initial PDF $P(X_S(0))$ (which is assumed to be known). These state vectors are called particles and are denoted as $X_S(k|k)$ (k=1,2,…,N). At each time step, we propagate the particles to the next time step using the process

$$X_s(k+1/k) = f(X_s(k-1/k), w(k+1))$$

equation. (k=1,2,…,N). (10)

where each w(k+1) noise vector is randomly generated on the basis of the known PDF of w(k). After we receive the measurement at time k, we compute the conditional relative likelihood of each particle**Error! Reference source not found.**. That is, we evaluate the PDF $P(Z(k)|X_S(k+1$**Error! Reference source not found.**. This can be done if we know the nonlinear measurement equation and the PDF of the measurement noise. For example, if an m-dimensional measurement equation is given as **Error! Reference source not found.** and **Error! Reference source not found.** then a relative likelihood q(k), that the measurement is equal to a specific measurement z given the premise that **Error! Reference source not found.** is equal to the particle **Error! Reference source not found.** can be computed as follows [18]. **Error! Reference source not found.**

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$$\sim \frac{1}{(2\pi)^{m/2}|R|^{1/2}} \exp\left(\frac{-[z-h(X_s(k+1|k))]^T R^{-1}[z-h(X_s(k+1|k))]}{2}\right) \quad (11)$$

The ~ symbol in the above equation means that the probability is not really given by the expression on the right side, but the probability is directly proportional to the right side. So if this equation is used for all the particles, **Error! Reference source not found.**, then the relative likelihoods that the state is equal to each particle will be correct. Now we normalize the relative likelihoods obtained in eqn. (11) as follows.

$$q(k) = q(k) \Big/ \sum_{i=1}^{N} q(i)$$ **Error! Reference source not found. Error! Reference source not found.** (12)

Now we resample the particles from the computed likelihoods and a new set of particles that are randomly generated on the basis of the relative likelihoods q(k).

2.3 Particle Filtering Combined with other Filters

One approach that has been proposed for improving particle filtering is to combine it with another filter such as the EKF, UKF or MGBEKF [18]. In this approach, each particle is updated at the measurement time using the EKF, UKF or MGBEKF and then resampling (if required) is performed using the measurement. This is like running a bank of N Kalman filters (one for each particle) and then adding a resampling step after each measurement. After **Error! Reference source not found.** is obtained, it can be refined using the EKF, UKF or MGBEKF measurement-update equations. In this paper PF is combined with the MGBEKF.**Error! Reference source not found.** is updated to **Error! Reference source not found.** according to the following MGBEKF equations [18]. **Error! Reference source not found.**(13)

Error! Reference source not found.Error! Reference source not found.Error! Reference source not found. (14)

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Error! Reference source not found.(15) **Error! Reference source not found.** $P(k+1|k+1)_i =$ **Error! Reference source not found.**

Error! Reference source not found. Error! Reference source not found.Error! Reference source not found. (16)

where **Error! Reference source not found.** is Kalman gain, **Error! Reference source not found.** is a priori estimation error covariance for the ith particle and g (.) is modified gain function. g(.) is given by

$$g = \begin{bmatrix} 0 & 0 & \frac{\cos B_m}{\hat{R}_x \sin B_m + \hat{R}_y \cos B_m} & \frac{-\sin B_m}{\hat{R}_x \sin B_m + \hat{R}_y \cos B_m} \end{bmatrix} \quad (17)$$

Since true bearing is not available in practice, it is replaced by the measured bearing to compute the function g (.).

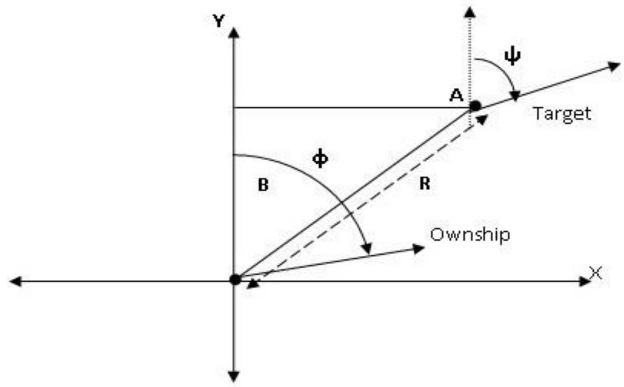
Resampling: In every update of PFMGBEKF, it is monitored to decide whether resampling of particles in respect of target state vector and its covariance matrix is required or not. Resampling is required when the effective sample size, $N_{eff} < N/3$ [18], where

Error! Reference source not found. $$N_{eff} = 1 \Big/ \sum_{i=1}^{N} q_i^2 \quad (18)$$

whenever re-sampling is required, the following procedure based on weights of particles is adopted. In this method, weights are sorted in descending order. The corresponding original indexes prior to sorting are remembered. Then replication of particles (both the state and covariance matrices) is carried out in proportion to the weights of the particles starting with the particle with maximum weight age. This procedure is repeated for the particle with the next maximum weight age. This process is continued till all the particle positions are filled up. This method is close to the method suggested by B. Ristick, S. Arulampalam and N. Gordon [17].

2.4 Closest path of approach

Let us assume that a target and ownship are moving at predefined constant velocities. At certain point of time these vehicles move through a point at which minimum distance will be there between them. This minimum distance is called Closest Path of Approach (CPA). Once torpedo motion parameters are estimated using PFMGBEKF, CPAs are calculated for all possible ownship evasive courses (say 0 to 360 in step of 1 deg). Ownship will do evasive manoeuvre in the course at which maximum CPA is generated. CPA is calculated as follows. It is assumed that target motion parameters and ownship parameters are known. Initially ownship is at the origin. Let the ownship & target courses be ϕ and ψ respectively. The distance between target and ownship



positions at time t can be derived as follows

Fig. 1. Ownship and target encounter.

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Error! Reference source not found. (20)

where V_t and V_0 are the speeds of target and ownship respectively. To simplify the eqn. (20), let

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Error! Reference source not found. then eqn.(19) & eqn. (20) become **Error! Reference source not found.** (21) **Error! Reference source not found.** (22) The distance R_t between ownship and target is given by **Error! Reference source not found.**

By differentiating **Error! Reference source not found.** with respect to time and equating it to zero,

$\frac{d}{dt}(R_t^2) = 2(m^2+n^2)t + 2(mp+nq) = 0.$ **Error! Reference source not found.** (23)

For a particular value of t say **Error! Reference source not found.** equation (23) can be written as

$t_m = \frac{(pm+qn)}{m^2+n^2}$ **Error! Reference source not found.** (24)

At this stage, taking second derivative, we have,

$\frac{d^2}{dt^2}(R_t^2) = 2(m^2+n^2)$**Error! Reference source not found.** (25)

and it is always greater than zero. Hence t_m gives minimum time at which the distance R is minimum. If t_m <= 0, it implies that present range is CPA and time to reach CPA point is zero. If**Error! Reference source not found.** , substituting the value of **Error! Reference source not found.** in eqn. (21), we will get **Error! Reference source not found.** as follows:

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Error! Reference source not found. (26)

Since**Error! Reference source not found.**, equation (26) can be modified as follows:

Error! Reference source not found. (27)

R_t is nothing but CPA. So

Error! Reference source not found. (28)

3 Implementation And Simulation

+For the implementation of the algorithm, the initial estimate of target state vector is chosen as follows. As only bearing measurements are available, it is not possible to guess the velocity components of the target. So these components are each assumed as 15 m/sec, which are close to the realistic speed of the torpedo. The range of the day, say 10000 meters, can be utilized in the calculation of initial position components of the torpedo as follows

Error! Reference source not found. (29) It is assumed that the initial estimate, X(0|0) is uniformly distributed. Then the elements of initial covariance diagonal matrix can be written as

Error! Reference source not found. (30)

As PF is combined with MGBEKF, 1000 particles (almost similar performance is achieved with 10000 particles) are used to estimate target motion parameters.

The measurement interval is assumed to be one second. It is also assumed that TA maximum auto detection range limit is 10000 meters. Estimation of torpedo motion parameters is stopped when the range is 500 meters. Maximum ownship speed is 11 m/sec. Ownship turning rate is considered 1 deg/sec. It is assumed that measurements are corrupted with one degree r.m.s error of Gaussian distribution. All angles are considered with respect to True North 0 to 360 degrees, clockwise positive. For the purpose of presentation, three scenarios as shown in Table 1 are considered for evaluation of the algorithm. The results obtained for the scenarios 1 to 3 are shown in Fig. 2 to 4 respectively. The estimated solution is said to be converged when

a. error in the range estimate <= 20% of the actual range
b. error in the course estimate <= 5 degs.
c. error in the speed estimate <= 4 knots.

The convergence time to obtain all the target motion parameters with the required accuracy for each scenario is shown in Table 1. The ownship evasive manoeuvre for each scenario is based on CPA. As it is straight forward to find out maximum CPA using eqn. (28), CPA results are not presented in the paper.

Table 1

S No	Initial Range (meters)	Initial Bearing (deg)	Target Speed (m/sec)	Target Course (deg)	Ownship Speed (m/sec)	Ownship Course (deg)	Convergence time (sec)
1	4500	90	15.45	293 (0°)	6.18	0°	145
2	6000	270	15.45	66.42 (0°)	6.18	0°	128
3	5000	320	15.45	125 (0°)	6.18	0°	124

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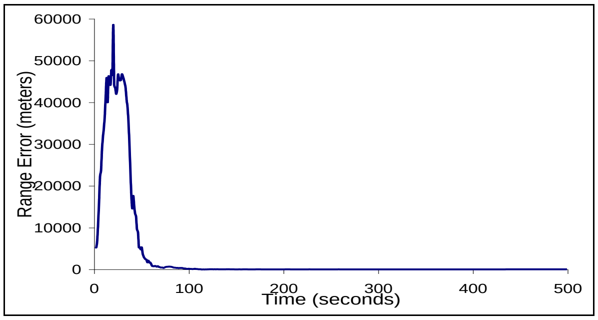


Fig. 2(a). Error in range estimate

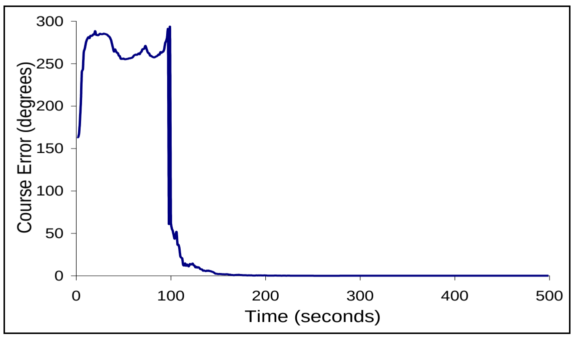


Fig. 2(b). Error in course estimate

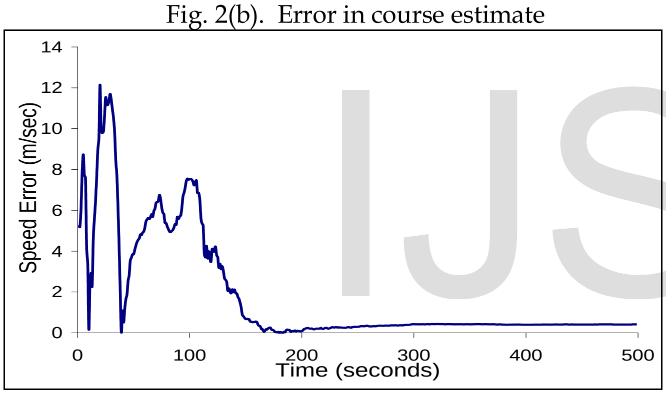
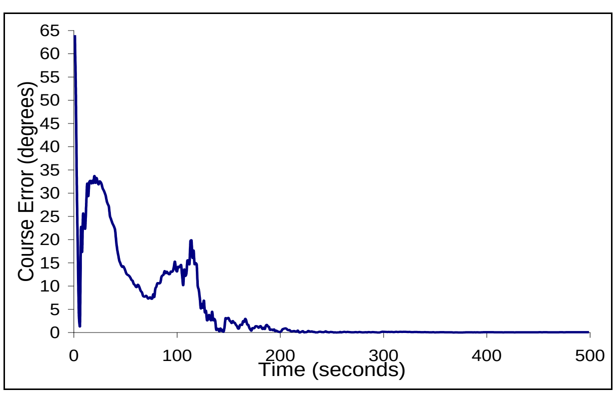


Fig. 2(c). Error in Speed estimate



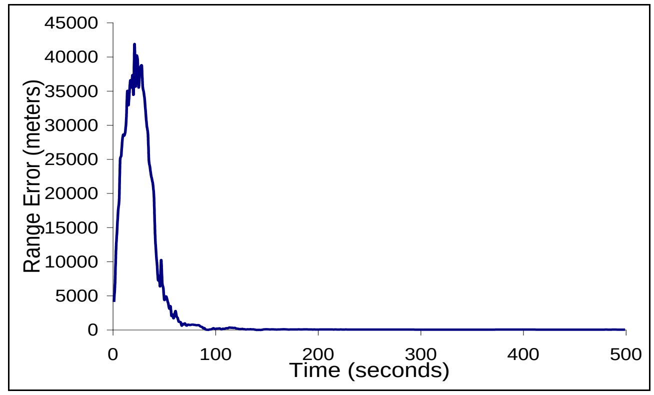


Fig. 3(a). Error in range estimate

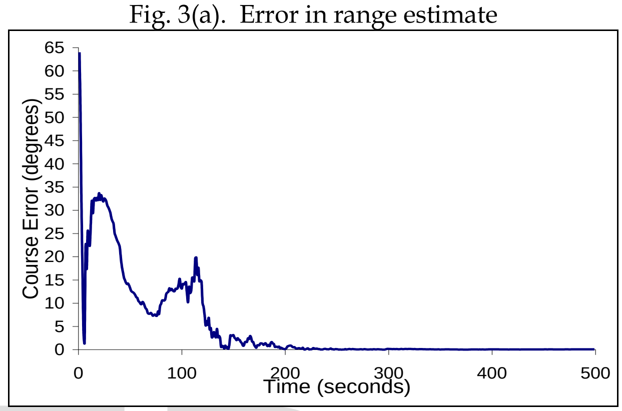


Fig. 3(b). Error in course estimate

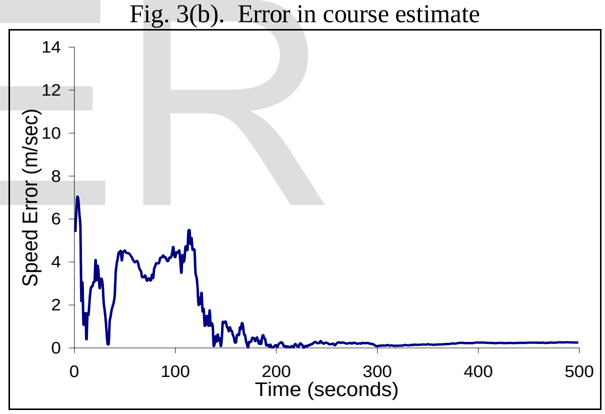


Fig. 3(c). Error in speed estimate

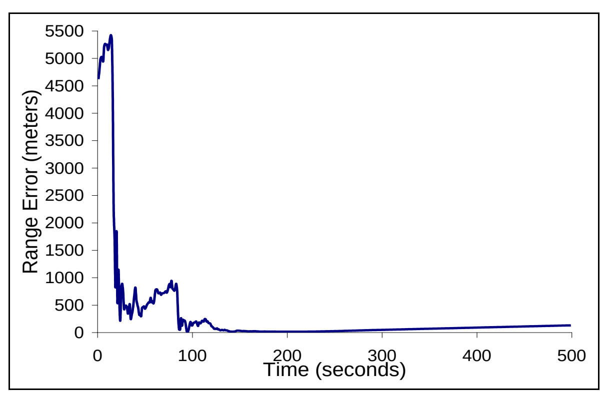


Fig. 4(a). Error in range estimate

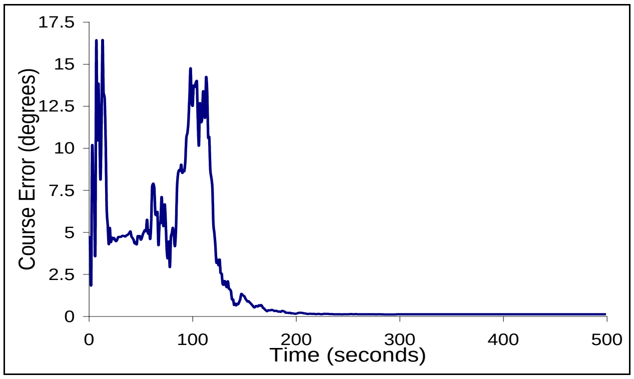


Fig. 4(b). Error in course estimate

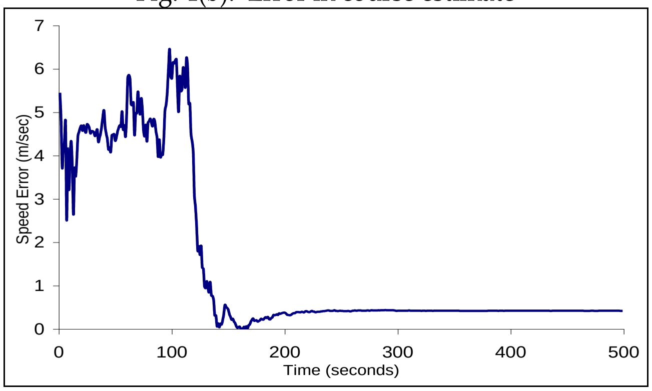


Fig. 4(c). Error in speed estimate

4 Limitations of the Algorithm

Angle on Target Bow (ATB) is the angle between the target course and line of sight. When ATB is more than 60^0, the distance between the target and ownship increases, as time increases and the bearing rate decreases substantially with the increase in number of samples. In such situation, it is very difficult to track the target. Also, the algorithm cannot provide good results when the measurement noise is more than 1 degree r.m.s. In general, these two situations are constraints to any type of filtering technique.

5 Conclusion

Particle Filter (which is useful for nonlinear and non-Gaussian applications) combined with MGBEKF is proposed to estimate target motion parameters in passive target tracking. The performance of the PFMGBEKF is greatly superior to the standard extended Kalman filter. In this paper, tracking of torpedo using towed array measurements is explored. Ownship safety manoeuvre is used for observablility of the process. CPA method uses the estimated torpedo motion parameters to find out ownship evasive manoeuvre. Extensive simulation is carried out and the results are found to be consistent. For the purpose of presentation, results of three typical scenarios are presented.

Appendix A

In safety manoeuvre algorithm, it is observed whether the torpedo is on port side (sign of torpedo is negative) or starboard side (sign of torpedo is positive). If the absolute value of the relative bearing is less than 30^0, then evasive manoeuvre is equal to measured bearing plus (sign of torpedo side)*30^0. If the absolute value of relative bearing is greater than or equal to 70^0, then evasive manoeuvre is equal to 180^0 + measured bearing +(sign of torpedo side)*30 deg.

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